Heat transport by laminar boundary layer flow with polymers

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Motivated by recent experimental observations, we consider a steady-state Prandtl-Blasius boundary layer flow with polymers above a slightly heated horizontal plate and study how the heat transport might be affected by the polymers. We discuss how a set of equations can be derived for the problem and how these equations can be solved numerically by an iterative scheme. By carrying out such a scheme, we find that the effect of the polymers is equivalent to producing a space-dependent effective viscosity that first increases from the zero-shear value at the plate then decreases rapidly back to the zero-shear value far from the plate. We further show that such an effective viscosity leads to an enhancement in the drag, which in turn leads to a reduction in heat transport.

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I. INTRODUCTION

It has been known for more than 60 years that adding polymers into turbulent wall-bounded flows can reduce the friction drag significantly (see, for example, [1, 2] and references therein). This effect of polymer additives on mass transport has been studied extensively during the past 60 or so years. On the other hand, the effect of polymers additives on heat transport is much less studied. Recently, an experimental study reported [3] that for turbulent Rayleigh-Bénard (RB) convection of water, confined within a cylindrical cell heated below and cooled on top, adding polymers to the flow reduces the heat transport. In turbulent RB convection within a container of given shape, the flow state is characterized by two dimensionless parameters: the Rayleigh number (Ra) and the Prandtl number (Pr), where Ra measures the size of the thermal forcing and Pr is the ratio of the kinematic viscosity to the thermal diffusivity of the fluid. Moreover, there is an exact balance [4] between the heat transport and the energy and thermal dissipation rates. The energy and thermal dissipation rates can be decomposed as sums of contributions from the bulk of the flow and from the boundary layers near the top and bottom plates [5]. The experimental study reported [3] was conducted at a Ra of the order of 10^{10}. At such a moderate Ra, the energy and thermal dissipation rates are mostly due to the contribution from the boundary layers [5]. This suggests that the observed reduction in heat transport is likely to be an effect of the polymers interacting with the boundary layer flow in turbulent RB convection. Experimental measurements [6] indicated that the average velocity and temperature boundary layer profiles in turbulent RB convection at moderate Ra could be described by those profiles in the steady-state Prandtl-Blasius boundary layer flow [7, 8] above a slightly heated flat plate.

Motivated by these experimental observations, we study a steady-state Prandtl-Blasius boundary layer flow with polymers near a slightly heated plate and focussed particularly on the possible effect of the polymer additives on the heat transport. Physically, we can think of this boundary layer flow as the flow near the bottom plate of the convection cell in turbulent RB convection.

This paper is organized as follows. In Section II, we formulate the problem and set up the equations of motion for the system. We discuss how the set of equations can be solved numerically by an iterative scheme in Section III. After solving the problem using such a scheme, our results show that the effect of the polymers is equivalent to producing a space-dependent effective viscosity that first increases from the zero-shear value at the plate then decreases rapidly back to the zero-shear value far from the plate. We further show that such an effective viscosity would lead to an enhancement in the friction drag and a reduction in heat transport. We shall present and discuss our results in Section IV. Finally, we shall summarize and conclude in Section V.

II. THE PROBLEM

For the Prandtl-Blasius boundary layer flow above a large flat plate, the velocity equation is:

$$v_x \partial_x v_x + v_y \partial_y v_x = \nu \partial_{yy}^2 v_x \qquad (1)$$

Here x denotes the direction along the plate, y denotes the direction away from the plate, and ν is the kinematic viscosity of the fluid. Both v_x and v_y vanish at the plate and v_x merges with the uniform mainstream velocity U far away from the plate. The crucial point about Eq. (1) is that the viscous term is balanced against the nonlinear advection term and that the flow changes more rapidly away from the plate than along the plate such that $\partial_y \gg \partial_x$. The latter condition is satisfied for flows with large Reynolds number.

Introducing the variable

$$\xi \equiv \sqrt{\frac{U}{\nu x}} y \tag{2}$$

and the stream function

$$\Psi(x,y) \equiv \sqrt{\nu x U} \phi(\xi) \tag{3}$$

such that

$$v_x = \partial_y \Psi \; ; \qquad v_y = -\partial_x \Psi \tag{4}$$

we obtain the famous Blasius equation [7]

$$2\phi_{\xi\xi\xi} + \phi\phi_{\xi\xi} = 0 \tag{5}$$

showing that the velocity profile is self-similar in the dimensionless variable ξ at different position x along the plate. Here ϕ_ξ denotes $\partial_\xi \phi$. The boundary conditions are

$$\phi(0) = \phi_\xi(0) = 0 \; ; \qquad \phi_\xi(\infty) = 1 \tag{6}$$

as $v_x \to U$ when it is far away from the plate. The plate is slightly heated at a temperature T_1 above the ambient temperature T_0 far away from the plate. Writing the temperature field as

$$T(x,y) = T_0 + (T_1 - T_0)\theta(\xi) \tag{7}$$

[Relating to turbulent RB convection, T_0 can be taken as the temperature at the center of the cell and thus the total temperature difference across the cell is $2(T_1 - T_0)$]. Then θ satisfies the equation

$$2\theta_{\xi\xi} + \phi\theta_\xi \mathrm{Pr} = 0 \tag{8}$$

where $\mathrm{Pr} = \nu/\kappa$ and κ is the thermal diffusivity of the fluid, and the boundary conditions are:

$$\theta(0) = 1 \; ; \qquad \theta(\infty) = 0 \tag{9}$$

We want to investigate the effect of polymers on heat transport in this laminar Prandtl-Blasius flow. The polymers produces an additional stress in the momentum equation of the fluid. This polymer stress $\mathcal{T}_{ij}$ depends on the amount of stretching of the polymers and is thus a function of the dimensionless conformation tensor R_{ij} of the polymers. Let the vector $\vec{d}$ denote the polymer end-to-end distance and ρ_0 be the polymer radius in the unstretched regime, then R_{ij} represents the average over many (N) polymers in a small region around the point (x,y) of the product $d_i d_j/\rho_0^2$, i.e. $R_{ij} = N^{-1}\Sigma d_i d_j/\rho_0^2$. In the simplest Oldroyd-B model of polymers [9],

$$\mathcal{T}_{ij} = \frac{\nu_p}{\tau}(R_{ij} - \delta_{ij}) \tag{10}$$

where ν_p is the polymer contribution to the viscosity of the solution at zero shear and τ is the relaxation time of the polymers. Thus in the presence of polymers, the equation of motion for the velocity field is modified by an additional stress that depends on R_{ij}. By employing the same ideas leading to the Blasius equation [Eq. (1)] we have

$$v_x \partial_x v_x + v_y \partial_y v_x = \nu \partial_{yy}^2 v_x + \partial_y \left[\frac{\nu_p}{\tau} R_{xy} \right] \tag{11}$$

It has been shown that the relaxation time of the polymers can be significantly increased by the stretching of the polymers [10]. To model this effect, we let

$$\tau = \tau_0 \left(\frac{1 + aR}{1 + a} \right) \tag{12}$$

where τ_0 is the bare Zimm relaxation time, $R \equiv (R_{xx} + R_{yy})^{1/2}$, and $a > 0$ is a parameter. When there is no stretching, $R = 1$ and τ reduces back to τ_0. When there is stretching, $R > 1$ and $\tau > \tau_0$.

In the presence of polymers, the transformation using ξ does not lead to a similarity solution in general in that explicit appearance of x remains in the equation for ϕ. This is known in the literature. Similarity solution has been obtained in some special cases with certain velocity or temperature boundary conditions [11, 12] that might not have direct physical relevance. Here to circumvent this difficulty, we recall that the Prandtl-Blasius flow is meant to be applicable when x is large (such that $\partial_y \gg \partial_x$). Thus we make the following approximations:

$$v_y \approx \frac{1}{2}\sqrt{\frac{\nu_0 U}{L}}(\xi\phi_\xi - \phi) \tag{13}$$

$$\partial_x \approx -\frac{\xi}{2L}\frac{d}{d\xi} \; ; \qquad \partial_y \approx \sqrt{\frac{U}{\nu_0 L}}\frac{d}{d\xi} \tag{14}$$

by putting $x = L$, the length of the (long) plate, and replacing ν in ξ [Eq. (2)] by $\nu_0 = \nu + \nu_p$, where ν_0 is the total viscosity of the polymer solution at zero shear. That is, in the presence of polymers, we have

$$\xi = \sqrt{\frac{U}{\nu_0 x}} y \qquad \text{with polymers} \tag{15}$$

With these approximations, the scaling transformation of ξ leads to a similarity solution. The resulting modified Blasius equation is:

$$-\frac{1}{2}\phi\phi_{\xi\xi} = (1-\gamma)\phi_{\xi\xi\xi} + \frac{\gamma}{\mathrm{Wi}\sqrt{\mathrm{Re}}}\frac{d}{d\xi}\left[\frac{(1+a)R_{xy}}{1+aR}\right] \tag{16}$$

where the Weissenberg number (Wi) and the Reynolds number (Re) are defined as

$$\mathrm{Wi} \equiv \frac{\tau_0 U}{L} \; , \qquad \mathrm{Re} \equiv \frac{UL}{\nu_0} \tag{17}$$

and $\gamma \equiv \nu_p/\nu_0$ is a function of the polymer concentration. As usual in the Prandtl-Blasius approximation, all terms of the order of $1/\mathrm{Re}$ are nelglected in Eq. (16).

We would like to study how the heat transport is affected by the polymers. In turbulent RB convection, it

is common to measure the heat flux Q in terms of the dimensionless Nusselt number (Nu), which is the ratio of Q to that when there is only conduction, defined by

$$\text{Nu} = \frac{Q}{2k(T_1 - T_0)/H} = \frac{\langle -\frac{\partial T}{\partial y}\big|_{y=0}\rangle_A}{2(T_1 - T_0)/H} \quad (18)$$

where k is the thermal conductivity of the fluid, H is the height of the convection cell, and $\langle\ldots\rangle_A$ is the average over the cross section of the cell. For the Prandtl-Blasius flow, taking $H = L$ and dropping the numerical factor, Nu can be estimated as

$$\text{Nu} = \sqrt{\frac{UL}{\nu_0}}\, [-\theta_\xi(0)] \quad (19)$$

To proceed, we must supplement Eq. (16) with a specific information on R_{xy}. In a fluid flow of velocity $\vec{v}$, the components of the dimensionless polymer end-to-end distance $l_i = d_i/\rho_0$, $i = x, y$ obey the differential equations:

$$\frac{dl_i}{dt} = -\frac{1}{2\tau}(l_i - l_{0i}) + l_j\partial_j v_i + \text{thermal noise} \quad (20)$$

where $l_{0x} = \cos\alpha$ and $l_{0y} = \sin\alpha$ and α is a random angle uniformly distributed in $[0, 2\pi]$. Neglecting the thermal noise, we can rewrite Eq. (20) for the two-dimensional Prandtl-Blasius flow as

$$-\frac{L}{U}\frac{dl_x}{dt} = \frac{(1+a)(l_x - l_{0x})}{2\text{Wi}(1+aR)} + \frac{\xi\phi_{\xi\xi}}{2}l_x - \sqrt{\text{Re}}\phi_{\xi\xi}l_y \quad (21)$$

$$-\frac{L}{U}\frac{dl_y}{dt} = \frac{(1+a)(l_y - l_{0y})}{2\text{Wi}(1+aR)} + \frac{\xi^2\phi_{\xi\xi}}{4\sqrt{\text{Re}}}l_x - \frac{\xi\phi_{\xi\xi}}{2}l_y \quad (22)$$

In order to obtain l_i as a function of ξ, we assume that each polymer follows the streamline of the flow such that $d\xi/dt = v_x\partial_x\xi + v_y\partial_y\xi = -(U/2L)\phi$, again using the approximations in Eqs. (13) and (14). Thus $(L/U)d/dt = -(\phi/2)d/d\xi$ and Eqs. (21) and (22) become

$$\phi\frac{dl_x}{d\xi} = \frac{(1+a)(l_x - l_{0x})}{\text{Wi}(1+aR)} + \xi\phi_{\xi\xi}l_x - 2\sqrt{\text{Re}}\phi_{\xi\xi}l_y \quad (23)$$

$$\phi\frac{dl_y}{d\xi} = \frac{(1+a)(l_y - l_{0y})}{\text{Wi}(1+aR)} + \frac{\xi^2\phi_{\xi\xi}}{2\sqrt{\text{Re}}}l_x - \xi\phi_{\xi\xi}l_y \quad (24)$$

The quantity R_{xy} is the average of $l_x l_y$ over all the polymers contained in a small volume centered near the point (x, y), over the angle α, and over the thermal noise. Such an average would depend on the precise distribution of the polymers, which is not obvious to obtain. Thus instead of performing such an average, we calculate l_x and l_y for one polymer for a fixed angle $\alpha = \pi/4$ such that $l_{0x} = l_{0y} = l_0 = 1/\sqrt{2}$, and estimate R_{xy} as some function of the calculated l_x and l_y. In an earlier short communication [13], we have shown that neglecting the thermal noise and in the limit of small Wi and $a = 0$, the averaging over the angle α gives $R_{xy} = [1 + h(\xi)]\phi_{\xi\xi}\text{Wi}\sqrt{\text{Re}}$

with $h(\xi) \to 0$ as $\xi \to \infty$. We expect this behavior of R_{xy} going like $\text{Wi}\sqrt{\text{Re}}\phi_{\xi\xi}$ far away from the plate is generically true. Moreover, far away from the plate, $R \to 1$ and $l_y \to l_0$. Thus we estimate R_{xy} as

$$R_{xy} = \text{Wi}\sqrt{\text{Re}}\left(\frac{1+aR}{1+a}\right)\phi_{\xi\xi} + (l_x - l_0)(l_y - l_0) \quad (25)$$

which has the desired asymptotic behavior far away from the plate. Defining

$$g(\xi) \equiv \frac{[l_x(\xi) - l_0][l_y(\xi) - l_0](1+a)}{\text{Wi}\sqrt{\text{Re}}[1 + aR(\xi)]\phi_{\xi\xi}(\xi)} \quad (26)$$

with $R(\xi) = [l_x^2(\xi) + l_y^2(\xi)]^{1/2}$, then

$$R_{xy} = \text{Wi}\sqrt{\text{Re}}\frac{(1+aR)}{(1+a)}\phi_{\xi\xi}(1+g) \quad (27)$$

Substituting Eq. (27) into Eq. (16), we have

$$2\phi_{\xi\xi\xi} + \phi\phi_{\xi\xi} + 2\gamma\frac{d}{d\xi}(g\phi_{\xi\xi}) = 0 \quad (28)$$

Comparing Eq. (28) with Eq. (5), it is obvious that the effect of the polymers is contained in the term with $g(\xi)$. Furthermore, this effect of the polymers is equivalent to producing a space-dependent viscosity $\nu_{\text{eff}}(\xi)$. Specfically, if we define ν_{eff} by

$$\frac{\nu_p}{\tau(\xi)}R_{xy}(\xi) \equiv \nu_{\text{eff}}(\xi)\partial_y v_x \quad (29)$$

and take

$$\nu_{\text{eff}}(\xi) = \nu_p[1 + g(\xi)] \quad (30)$$

then we obtain exactly Eq. (28) from Eq. (11). The total effective viscosity of the polymer solution is thus

$$\nu_{\text{tot}}(\xi) = \nu + \nu_{\text{eff}}(\xi) = \nu_0 + \nu_p g(\xi) \quad (31)$$

with the nontrivial effect of the polymers contained in the term $\nu_p g(\xi)$, which is a function of the polymer concentration.

Equations (23), (24), (26) and (28) should be solved consistently. With the obtained $\phi(\xi)$, Eq. (8) is solved to obtain $\theta(\xi)$ and from which Nu is obtained by using Eq. (19). We would like to compare this Nu in the presence of polymers to the reference value Nu_0 for a Newtonian fluid with the *same* kinematic viscosity ν_0 at the plate. To get the reference value Nu_0, we start with the Prandtl-Blasius velocity profile for a Newtonian fluid of kinematic viscosity ν_0, denoted as $\phi^{(B)}(\xi)$. This is just the solution to Eq. (5) with ξ given by Eq. (15) or Eq. (28) with $\gamma = 0$. With this $\phi^{(B)}$, we solve Eq. (8) and obtain the resulting temperature profile $\theta^{(B)}(\xi)$ at the same $\text{Pr} = \nu_0/\kappa$ as the polymer solution. Then

$$\text{Nu}_0 = \sqrt{\frac{UL}{\nu_0}}[-\theta_\xi^{(B)}(0)] \quad (32)$$

We are interested in the ratio

$$\frac{\text{Nu}}{\text{Nu}_0} = \frac{\theta_\xi(0)}{\theta_\xi^{(B)}(0)} \quad (33)$$
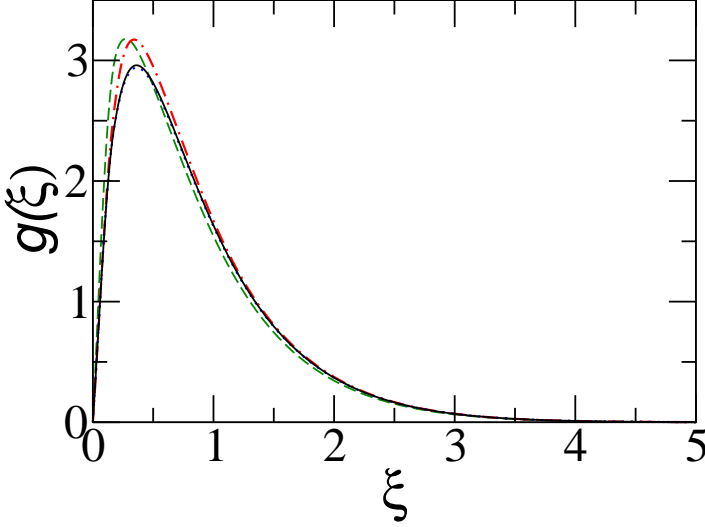
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FIG. 1: Fast convergence of the numerical iterative procedure. Result of $g(\xi)$ obtained after the first (dashed), second (dot-dashed), fifth (dotted) and 10th iteration (solid).

III. CALCULATIONS

We solve Eqs. (23), (24), (26) and (28) consistently by iteration at fixed values of Re and a. We start with $\phi = \phi^{(B)}$ in Eqs. (23) and (24) and solve for $l_x(\xi)$ and $l_y(\xi)$. Using the boundary condition that l_x and $l_y \to l_0$ as $\xi \to \infty$, we put $l_x = l_y = l_0$ at some large value of ξ, denoted as ξ_∞ (we use $\xi_\infty = 30$) and integrate backwardly until ξ equals to some finite ξ_0 close to zero. We cannot integrate forwardly in ξ because ϕ vanishes at $\xi = 0$ [see Eq. (6)]. With the calculated l_x and l_y, we obtain $g(\xi)$ for ξ between ξ_0 and ξ_∞ using Eq. (26). From Eq. (24), we see that $l_y(0) = l_0$ and thus $g(0) = 0$. Between $\xi = 0$ and ξ_0 we extrapolate $g(\xi)$ using a polynomial fit. We input this g into Eq. (28) to solve for an updated ϕ. Then we use this updated ϕ in Eqs. (23) and (24) to obtain an updated g. We repeat the procedure until convergence in both g and ϕ is achieved.

In Fig. 1, we show $g(\xi)$ obtained using this iterative procedure at Re= 4900, $a = 0.01$, Wi = 2.8 and $\gamma = 0.2$. Convergence is fast and achieved after only a few iterations. Using the converged ϕ, we obtain Nu for Pr=4.4 as discussed in Section II, and study the ratio Nu/Nu$_0$ for different values of the parameters.

There is a constraint on the possible values of the parameters imposed by the realizability of $l_x(0)$. From Eq. (23), we see that $l_x(0)$ satisfies the following equation:

$$\frac{1 + 1/a}{2\text{Wi}\sqrt{\text{Re}}\phi_{\xi\xi}(0)l_0}[l_x(0) - l_0] = \sqrt{l_x(0)^2 + l_0^2} + \frac{1}{a} \quad (34)$$

Thus the condition for Eq. (34) to have a finite real so-

lution for $l_x(0)$ is:

$$\left(1 + \frac{1}{a}\right)\frac{1}{\text{Wi}} > 2\sqrt{\text{Re}}\phi_{\xi\xi}(0)l_0 \quad (35)$$

As a result, for a given value of Re, the range of allowed values of Wi is smaller for larger values of a. Moreover, the allowed range of Wi is larger for a smaller value of Re. We have used two values of Re: 100 and 4900. For Re=4900, we take $a = 0.01$ and find that for $\gamma = 0.2$, the maximum allowed value of Wi is about 2.8. For Re=100, we take $a = 0.07$ and study different values of Wi up to 3.0 for the same value of γ. We then fix Wi=2.5, and study the effect of polymer concentration by varying the value of γ for Re=4900 and $a = 0.01$. We study also a few other values of the parameters to investigate the dependence of the results on the parameters.

IV. RESULTS AND DISCUSSIONS

We find that $g(\xi)$ increases from zero at the plate ($\xi = 0$) up to a certain maximum then decreases rather rapidly back to zero far away from the plate ($\xi \to \infty$). Whenever $g(\xi)$ is larger than zero, the viscosity of the polymer solution is enhanced compared to that of the solvent. We expect that such an increase in the viscosity gives rise to an enhancement of the friction drag, which would in turn result in a reduction of the horizontal velocity. Indeed, we find that the horizonal velocity, v_x^p for the flow with polymers is reduced compared to v_x^0, the horizontal velocity for the flow without polymers (see Fig. 2).

We study the change in drag by measuring directly the drag coefficient, defined by:

$$C = \frac{\nu_0\partial_y v_x\big|_{y=0}}{1/2U^2} = \frac{1}{2\sqrt{\text{Re}}}\phi_{\xi\xi}(0) \quad (36)$$

We are interested in the ratio

$$\frac{C}{C_0} = \frac{\phi_{\xi\xi}(0)}{\phi_{\xi\xi}^{(B)}(0)} \quad (37)$$

An enhancement in the friction drag implies a reduction of heat transport. To see this, we note that upon double integration of Eq. (8) by ξ, we obtain:

$$-\theta_\xi(0) = \frac{1}{\int_0^\infty dt\,\exp[-\frac{\text{Pr}}{2}\int_0^t \phi(s)ds]} \quad (38)$$

which tells us that Nu is a functional of ϕ. Define $\Phi(\xi) \equiv \int_0^\xi \phi(s)ds$, we can calculate δNu, the variation in Nu due to a variation in Φ:

$$\delta\text{Nu} = \frac{\text{Pr}}{2\sqrt{\text{Re}}}\text{Nu}^2\int_0^\infty \exp(-\text{Pr}\Phi(s)/2)\delta\Phi(s)ds \quad (39)$$

In the above expression, $\delta\Phi$ represents the variation of Φ due to the effect of the polymers. Since $v_x = U\phi_\xi$, the
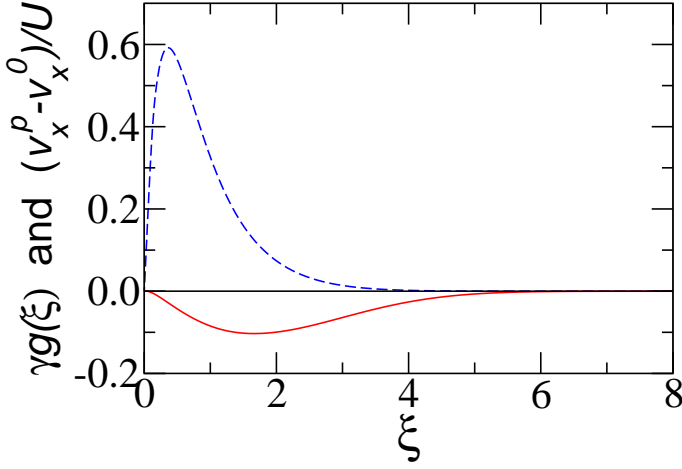
FIG. 2: Effective viscosity due to the polymers as represented by $\gamma g(\xi)$ (long dashed curve) and the difference in the horizontal velocity of the polymer solution from that of the pure solvent, $(v_x^p - v_x^0)/U$ (solid curve), as a function of ξ for Re=4900, $a = 0.01$, Wi=0.28, and $\gamma = 0.2$.

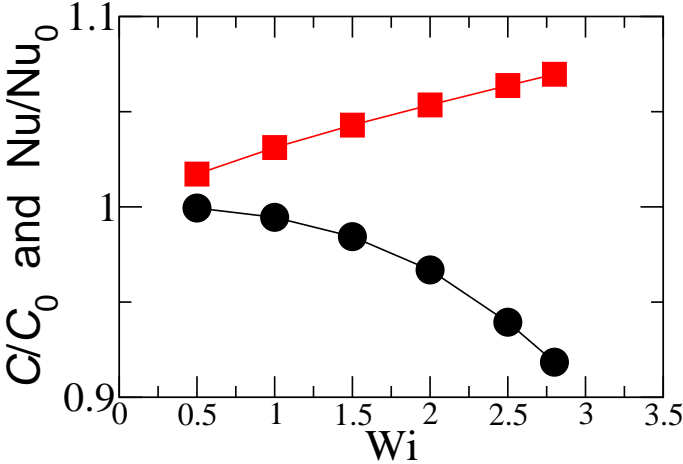FIG. 4: Dependence of the effective viscosity as measured by $\gamma g(\xi)$ on Wi at fixed polymer concentration for Re=4900, a=0.01 and $\gamma = 0.2$. From bottom to top, Wi increases from 0.5, 1.0, 1.5, 2.0. 2.5 to 2.8.

FIG. 3: C/C_0 (squares) and Nu/Nu_0 (circles) as a function of Wi for Re=4900, a=0.01 and $\gamma = 0.2$.

mass throughput in the x direction across a distance ξ is given by $U\phi(\xi)$, and thus a reduction in mass throughput implies an $\delta\Phi < 0$. Therefore, Eq. (39) shows that drag enhancement, i.e., reduction in the mass throughput implies a reduction of Nu. It is indeed found that $C/C_0 > 1$ while $Nu/Nu_0 < 1$ (see Fig. 3).

As shown in Fig. 3, the amount of drag enhancement and heat reduction increases with Wi. This is understood as the result of an increase in the effective viscosity with
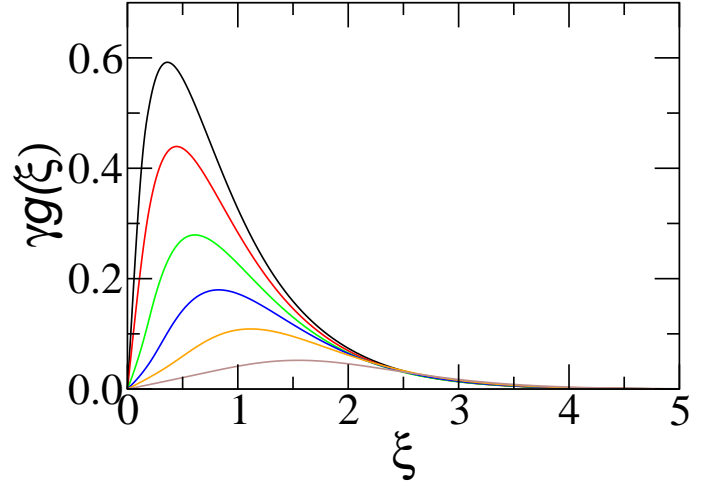
Wi. In Fig. 4, we show the dependence of $g(\xi)$ on Wi. It can be seen that the effective viscosity increases with Wi while the region in which the polymers are active (i.e., $g(\xi) > 0$) is approximately independent of Wi.

In Figs. 5 and 6, we show the dependence of the amount of drag enhancement (DE), $C/C_0 - 1$, and heat reduction (HR), $1 - Nu/Nu_0$, as a function of Wi at a fixed polymer concentration for the two different sets of values of Re and a studied. It can be seen that both effects are relatively modest. The effect is generally larger when Re is larger. Both %DE and %HR increases as Wi increases and for the range of Wi studied, they increase quadratically with Wi. Interestingly, for %DE the rate of increase decreases with Wi and this leads to the possibility of the saturation of the effect at large Wi. On the other hand, for %HR, the rate of increase increases with Wi thus the effect on heat reduction is larger than that but increases for %HR.

In Fig. 7, we fix Re=4900, a= 0.01, Wi=2.5 and show the extent of drag enhancement and heat reduction as a function of polymer concentration specified by γ. We see that the effect increases with γ as expected. Moreover, we see a possible saturation of the effect in the limit of the large polymer concentration. In Fig. 8, we show the dependence of $\gamma g(\xi)$ on γ. Again we see that γg increases with γ. It is this increase of the effective viscosity of the polymers with polymer concentration that leads to the increase in the extent of drag enhancement and heat reduction when polymer concentration increases.

We summarize the results for %DE and %HR obtained for different values of the parameters in Table 1. We see that the effect generally increases with a, Re, Wi and γ while the other parameters are kept fixed. We note that
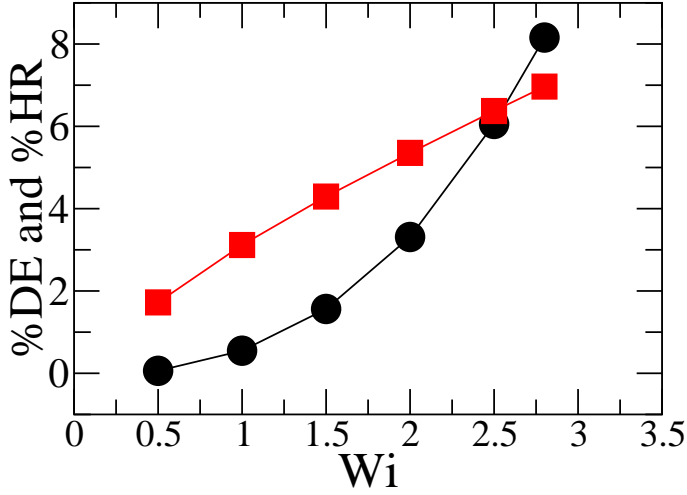
FIG. 5: Percentage of drag enhancement (% DE) (squares) and heat reduction (%HR) (circles) as a function of Wi at fixed $\gamma = 0.2$ for Re=4900 and a=0.01.
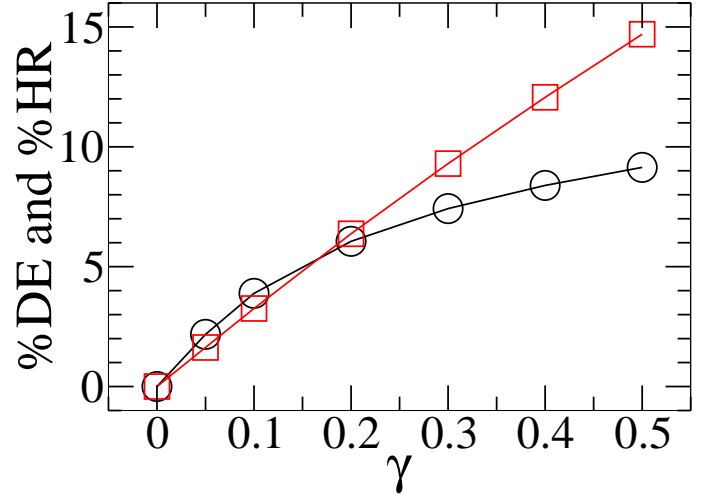

FIG. 7: Dependence of the effect on polymer concentration: %DE (squares) and %HR (circles) as a function of γ at fixed Wi=2.5.
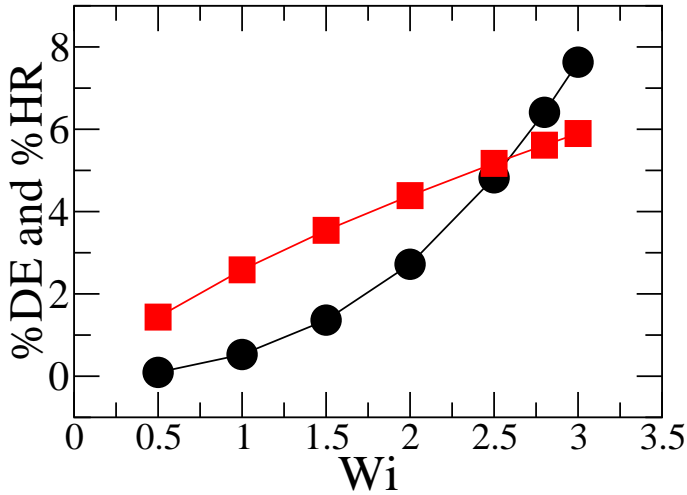

FIG. 6: %DE (squares) and %HR (circles) as a function of Wi at fixed $\gamma = 0.2$ for Re=100 and $a = 0.07$.

FIG. 8: Dependence of $\gamma g(\xi)$ on γ at fixed Wi=2.5. From bottom to top, γ increases from 0.1 to 0.5 in steps of 0.1.

drag enhancement and heat reduction are found even for $a = 0$ although the effect is relatively modest with only a few percentage of HR for $\gamma = 0.5$ and Wi=2.5. The effect increases with a and interestingly the increase in %HR is larger than that the increase in %DE when a is increased.
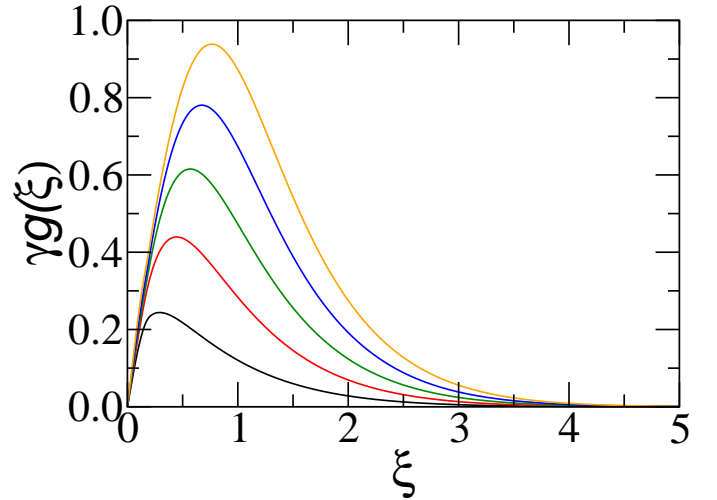
V. SUMMARY AND CONCLUSIONS

In this paper, we have studied the problem of heat transport in steady-state Prandtl-Blasius flow with polymers near a slightly heated plate. We have shown how a set of equations can be written for the problem and how this set of equations can be solved numerically by an iterative procedure. Our results demonstrate that the physical effect of the polymers is equivalent to produc-

Re	a	Wi	γ	% HR	% DE
100	0.005	3	0.2	2.1	4.5
100	0.05	3	0.2	5.0	5.4
100	0.2	0.5	0.2	0.2	1.6
100	0.4	0.5	0.2	0.5	2.0
100	0.6	0.5	0.2	0.9	2.4
100	0.7	0.5	0.2	1.3	2.5
100	0.07	1.0	0.2	0.5	2.6
100	0.07	1.5	0.2	1.4	3.5
100	0.07	2.0	0.2	2.7	4.4
100	0.07	2.5	0.2	4.8	5.2
100	0.07	2.8	0.2	6.4	5.6
100	0.07	3	0.2	7.6	5.9
4900	0.0	2.5	0.5	3.0	11.2
4900	0.0	2.5	0.2	1.5	4.7
4900	0.0	2.8	0.2	1.8	5.0
4900	0.1	0.3	0.2	0.7	2.2
4900	0.05	0.6	0.2	1.7	3.4
4900	0.005	3	0.2	4.2	6.1
4900	0.005	4	0.2	7.8	7.3
4900	0.005	5	0.2	12.1	8.4
4900	0.01	1.0	0.2	0.6	3.1
4900	0.01	1.5	0.2	1.6	4.3
4900	0.01	2.0	0.2	3.3	5.4
4900	0.01	2.5	0.2	6.1	6.3
4900	0.01	2.8	0.2	8.2	7.0
4900	0.01	2.5	0.05	2.2	1.6
4900	0.01	2.5	0.1	3.9	3.2
4900	0.01	2.5	0.3	7.4	9.3
4900	0.01	2.5	0.4	8.4	12.1
4900	0.01	2.5	0.5	9.1	14.7

TABLE I: Amount of drag enhancement and heat reduction for different values of parameters.

ing a space-dependent effective viscosity, which increases near the plate. Because of this increase in viscosity, drag is enhanced. We have shown that such a drag enhancement then leads to a reduction in heat transport in the Prandtl-Blasius flow with polymers. As discussed in Sec. I, in turbulent RB convection of Ra about 10^{10}, heat transport is dominated by contributions from the boundary layers and the mean velocity and temperature boundary layer profiles were found to be well-described by the Prandtl-Blasius profiles. Hence, our theory may explain the recent experimental observation of a reduction in heat transport in turbulent RB convection with polymer additives. In particular, an amount of about 10 % HR can be obtained with suitable parameters in our theory, and this amount of heat reduction is comparable to that observed in the experiment.

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